Can we infer rheological properties from live imaging data ?

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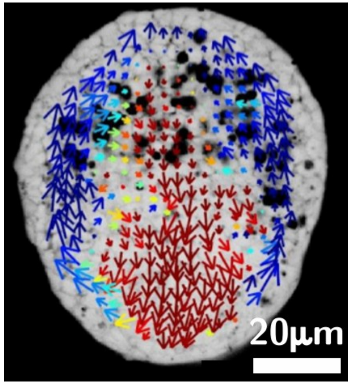
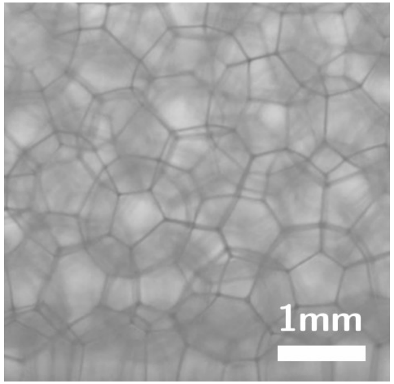
Context of the rheometry

Most of the fluid-like materials exhibit a non-Newtonian behavior



Biological
(tissues,blood, ...)

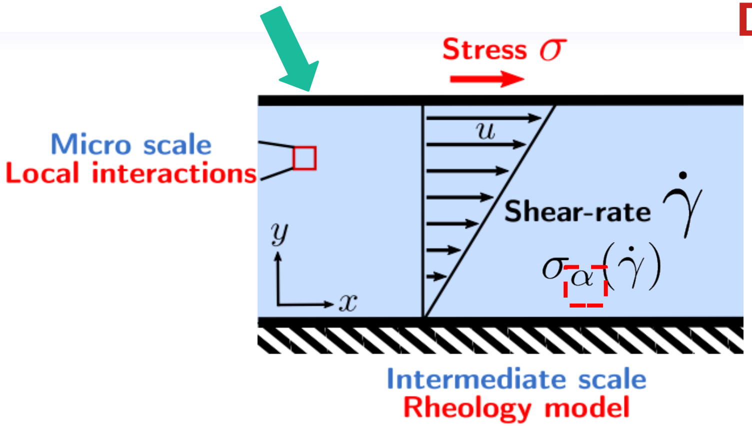


Industrial
(emulsions, foams, ...)

Geological
(sand, snow, mud, ...)

Context of the rheometry

Fluid like materials



Direct problem

3D generalisation

$$\dot{\gamma} \to S$$

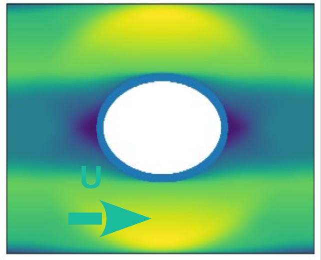
Stokes eq. (Visco-plastic)

Mass conservation

Momentum conservation

$$\begin{cases} \nabla \cdot U = 0 \\ \nabla P = \nabla \cdot (2\eta S(U)) \\ \eta = \eta_{\alpha}(\dot{\gamma}) \end{cases}$$

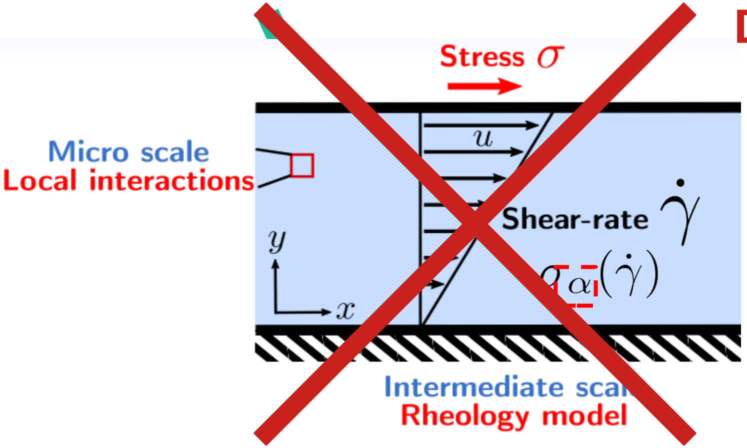
Horizontal velocity field



Herschel-Bulkley fluid : $\eta = \dfrac{\alpha_1}{\dot{\gamma}} + \dot{\gamma}^{\alpha_2 - 1}$

Context of the rheometry

Fluid like materials



~~Direct problem~~

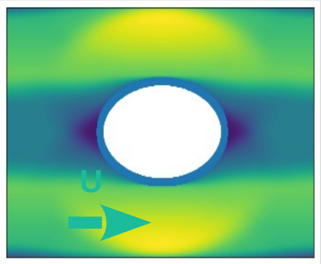
3D generalisation

$\dot{\gamma} \to S$

Fluid like materials velocity field

Stokes eq (Visco-plastic)

$$\begin{cases} \nabla \cdot U = 0 \\ \nabla P = \nabla \cdot (2\eta S(U)) \\ \eta = \eta_{\overset{?}{\boxed{\alpha}}}(\dot{\gamma}) \end{cases}$$



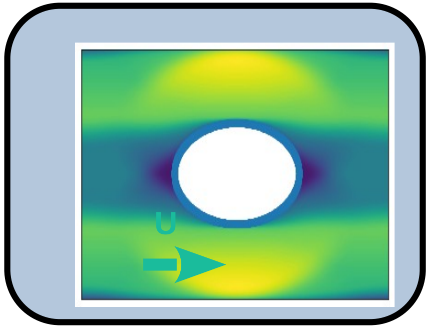
Herschel-Bulkley fluid : $\eta = \frac{\overset{?}{\boxed{\alpha_1}}}{\dot{\gamma}} + \dot{\gamma}^{\underset{?}{\boxed{\alpha_2}}-1}$

Common limits of the rheometry approach

- Generalisation to 3D/complex flows
- Some materials are not easy to put in a rheometer (sample volume, natural/biological fluids ...)

Context of the rheometry

Fluid like materials velocity field



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- Generalisation to 3D/complex flows
- Some materials are not easy to put in a rheometer (sample volume, natural/biological fluids ...)

Context of the rheometry

Stokes eq. (Visco-plastic)

$$\begin{cases} \nabla \cdot U = 0 \\ \nabla P = \nabla \cdot (2\eta S(U)) \\ \eta = \eta_{\boxed{\alpha}}(\dot{\gamma}) \end{cases}$$

? **?**

Herschel-Bulkley fluid : $\eta = \dfrac{\boxed{\alpha_1}}{\dot{\gamma}} + \dot{\gamma}^{\boxed{\alpha_2}-1}$

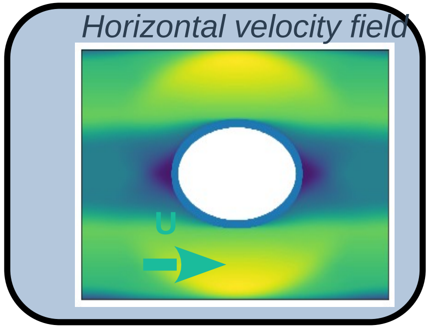
?

Inverse problem

Machine Learning algorithm

Physics Informed Neural Networks* (PINNs)

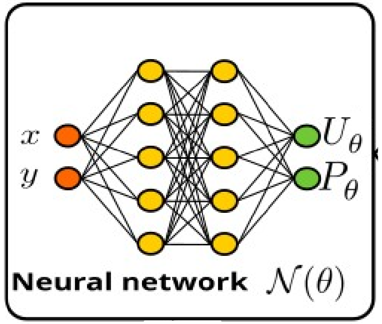
Fluid like materials velocity field



Find **optimal sparse parameters α** in order to solve the Stokes eq. for a **velocity field U** as close as possible from the experimental one.

**Raissi, et Al (2019).Journal of Computational physics.*

PINNs, how does it work ?



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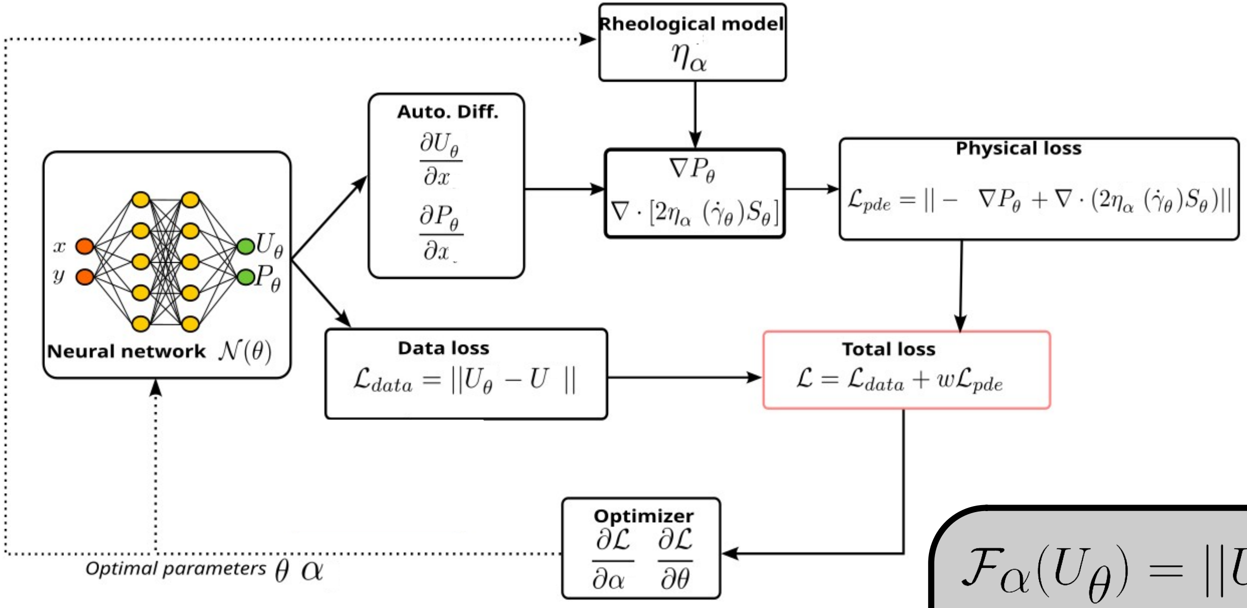


Optimisation problem

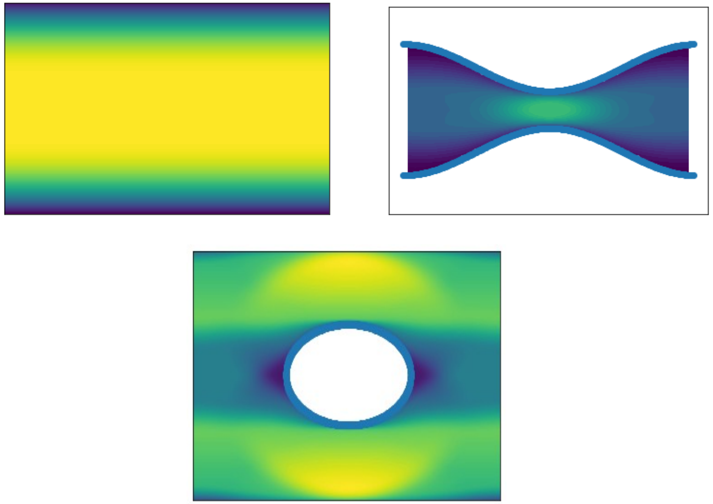
$$\mathcal{F}_\alpha(U_\theta) = ||U_\theta - U|| + w\,|| - \nabla P_\theta + \nabla \cdot \sigma_\alpha(U_\theta)||$$

Fidelity **Regularisation**

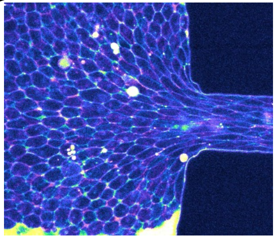
Find the set α and θ such that : $\mathcal{F}_\alpha(U_\theta) = 0$

Our approach

Test our approach on simulations , corresponding to relevant configurations for experimentalists

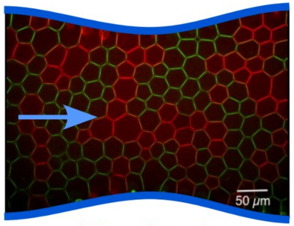


Numerical simulation



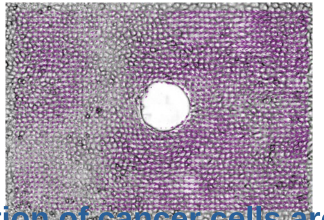
Microfluidic experiment

Sham Tlili



Wavy channel

Adapted from Golovkova (PhD thesis, 2021)



Migration of cancer cells around an obstacle,

Tlili, et al.. A microfluidic platform to investigate the role of mechanical constraints on tissue reorganization.

1st example : Uniform Channel HB

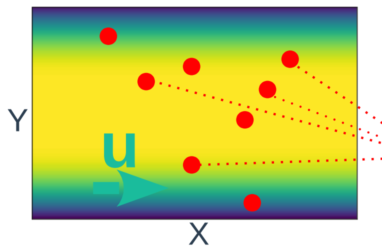
Physical Equation :
Hershel Buckley model

$$\begin{cases} \nabla \cdot U = & 0 \\ 0 = & -\nabla P + \nabla \cdot (2\eta(\dot{\gamma})S) \\ \eta = & \dfrac{\boxed{\alpha_0}}{\dot{\gamma}} + \dot{\gamma}^{\boxed{\alpha_1}-1} \end{cases}$$

Numerical Simulation

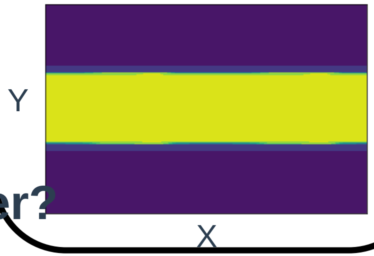
Numerical Simulation

Horizontal velocity field



Colocation points

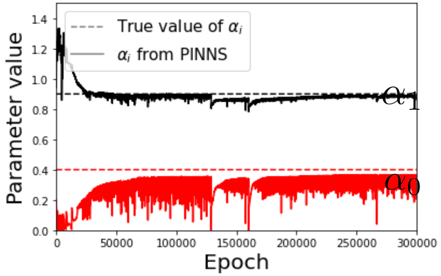
Viscosity field



PINNs

Output of the PINNs

Convergence history



Infered viscosity field

Up to how many parameters can we infer?

Is it robust to noise ?

What are the key data features giving an efficient inference ?

Carreau model, a 3 parameter inference

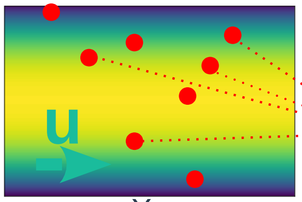
Physical Equation :
Carreau model

$$\begin{cases} \nabla \cdot U = 0 \\ 0 = -\nabla P + \nabla \cdot (2\eta(\dot{\gamma})S) \\ \eta = \boxed{\alpha_0} + (1 - \boxed{\alpha_0})(1 + (\boxed{\alpha_2}\dot{\gamma})^2)^{\frac{\boxed{\alpha_1} - 1}{2}} \end{cases}$$

Numerical Simulation

Numerical Simulation

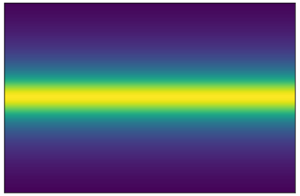
Horizontal velocity field



Colocation points

PINNs

Viscosity field



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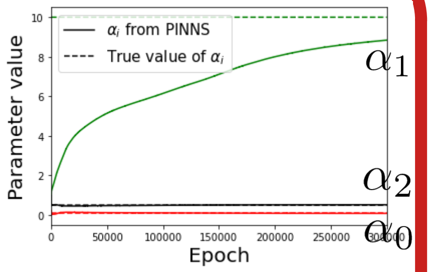
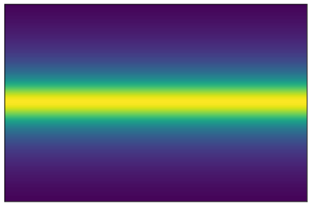


Infered Viscosity field



So far it works with a single velocity field with different models.

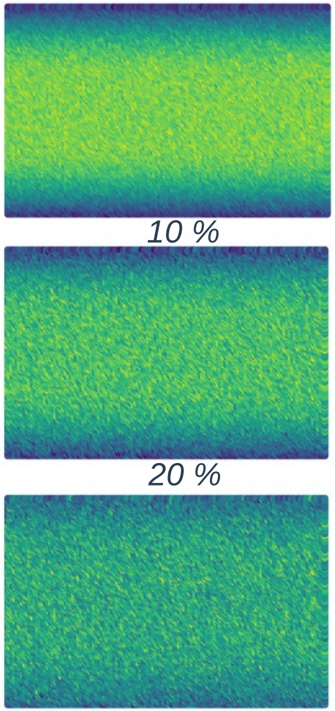
Noisy data : It is robust to noise ?

Numerical simulation

Horizontal velocity field

Noising

Horizontal noisy velocity field



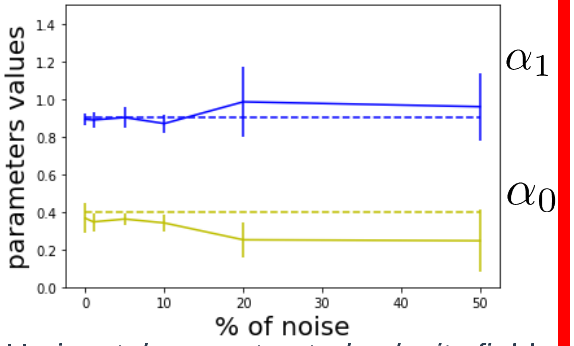
10 %

20 %

50 %

PINNs

Precision of the parameter depending of the level of noise

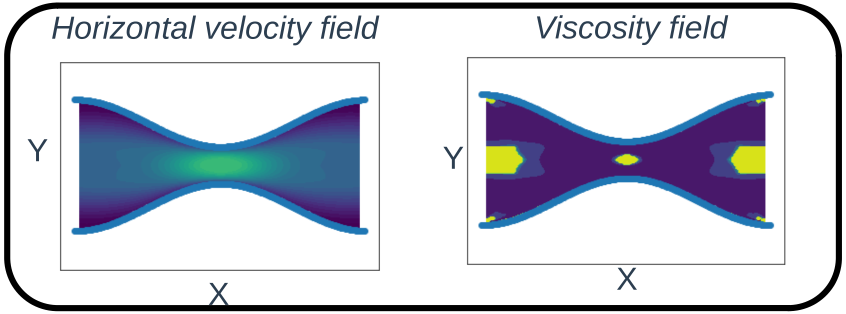


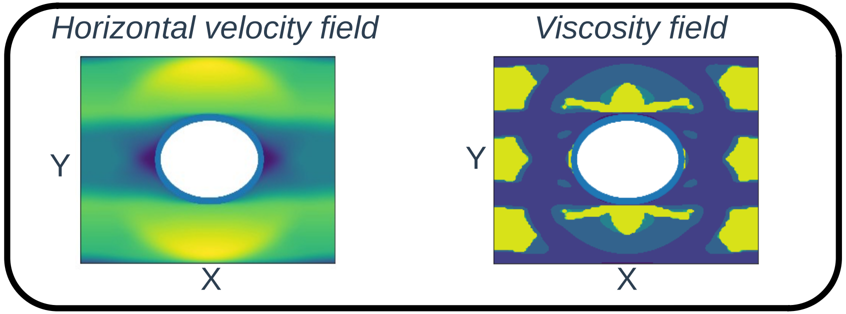
Horizontal reconstructed velocity field

The algorithm seems to be robust to noises.

The inference works with different flow geometries

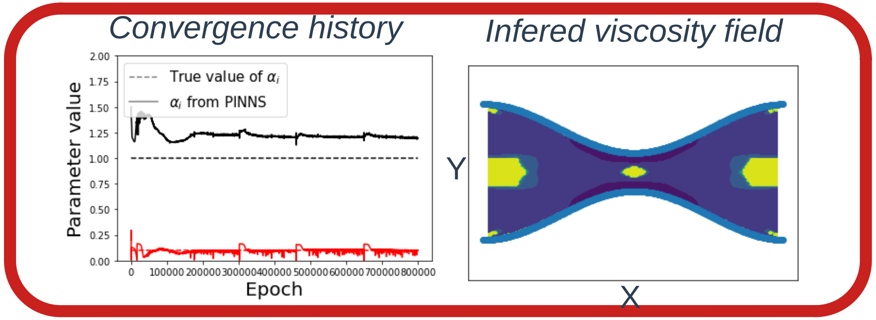
Numerical Simulation

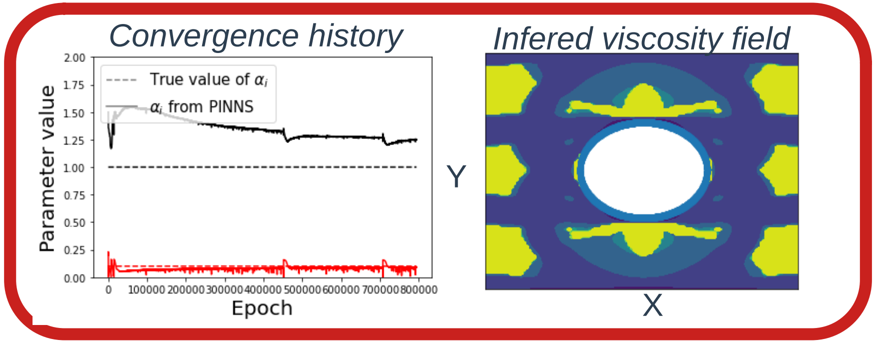




PINNs

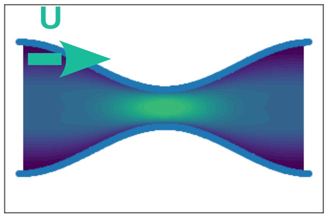
Output of the PINNs





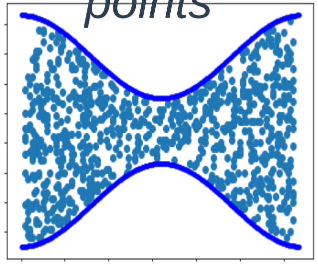
Importance sampling, what are the key informations ?

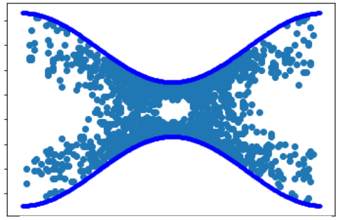
Horizontal velocity field



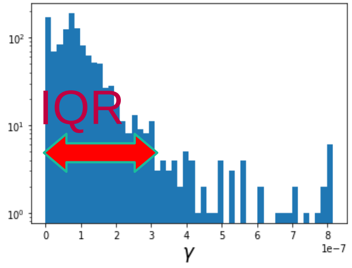
Colocation

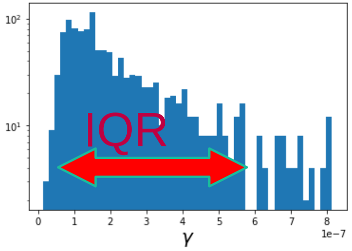
Sampling of the colocation points





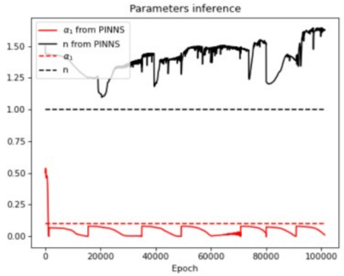
Histogram of $\dot{\gamma}$





PINNs

Convergence history



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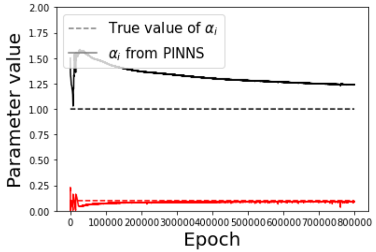

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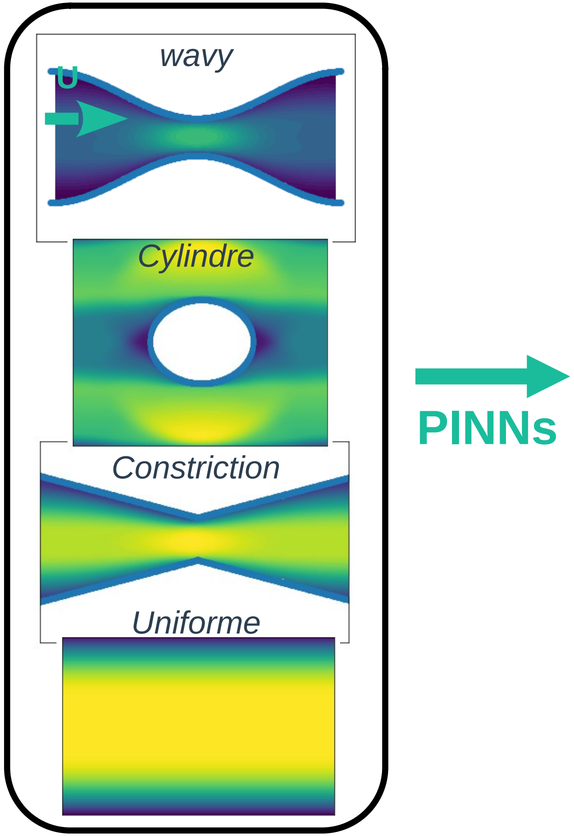
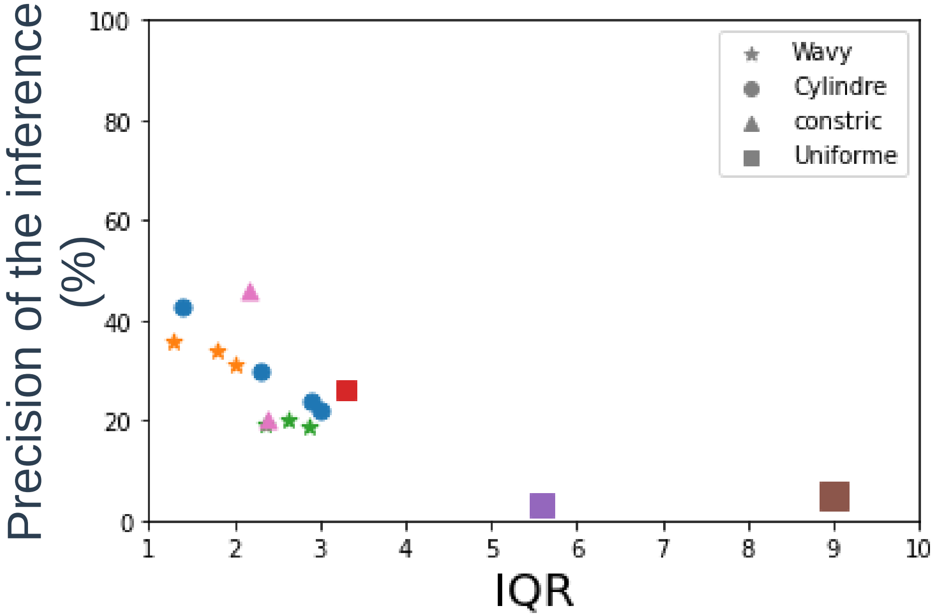




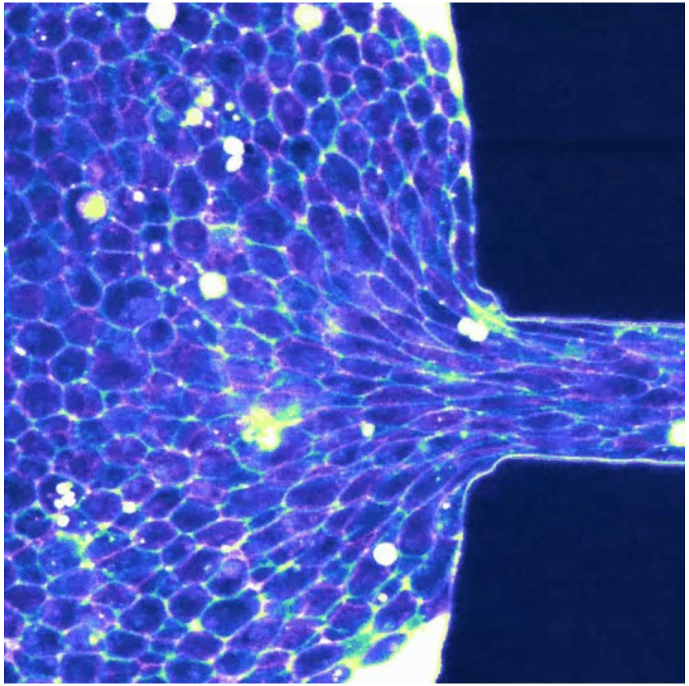
Key home message :

The more diversified the information is, the better the precision of the inference will be.

Long term Goal

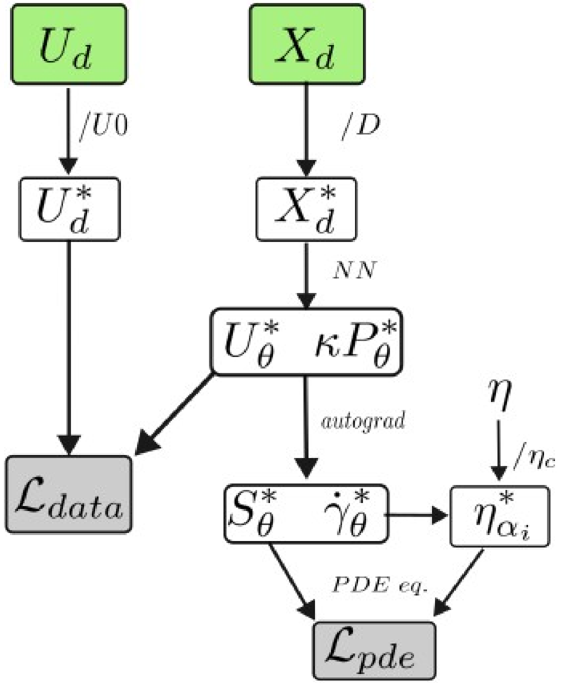
- **Use it on real data experiment.**
- **Include a large bank of rheological laws to allow** model selection**.**
- **Add visco-elastic models.**
- **What flow geometry is the most efficient to discriminate differente rheological models.**

Use it to infer the rheology of cellular tissues



Microfluidic experiment

Thanks for listening !


U_d
X_d
$/U0$
$/D$
U_d^*
X_d^*
NN
$U_\theta^* \quad \kappa P_\theta^*$
η
autograd
$/\eta_c$
$\mathcal{L}_{data}$
$S_\theta^* \quad \dot{\gamma}_\theta^*$
$\eta_{\alpha_i}^*$
PDE eq.
$\mathcal{L}_{pde}$